

Year 12 Mathematics Applications
Test 6 2018

Calculator Free
Project Networks and Assignment Problems

STUDENT'S NAME Solutions

DATE: Thursday 6th September

TIME: 50 minutes

MARKS: 45

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

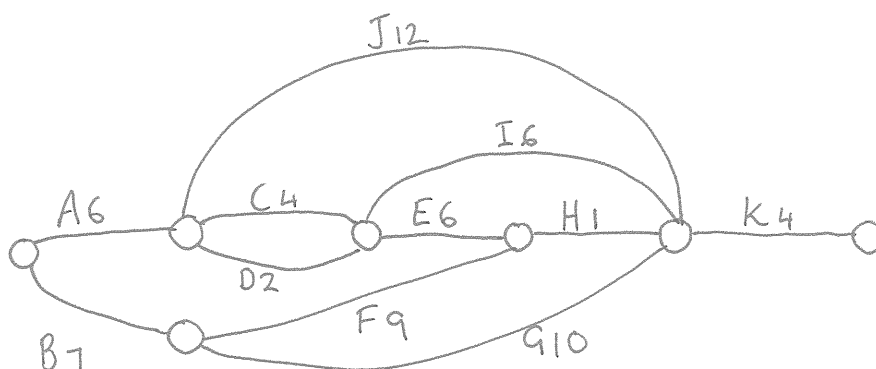
Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (5 marks)

The following table contains information relevant to a building project.

Task	Time for Task (Days)	Immediate predecessors
A	6	-
B	7	-
C	4	A
D	2	A
E	6	C,D
F	9	B
G	10	B
H	1	E,F
I	6	C,D
J	12	A
K	4	G,H,I,J

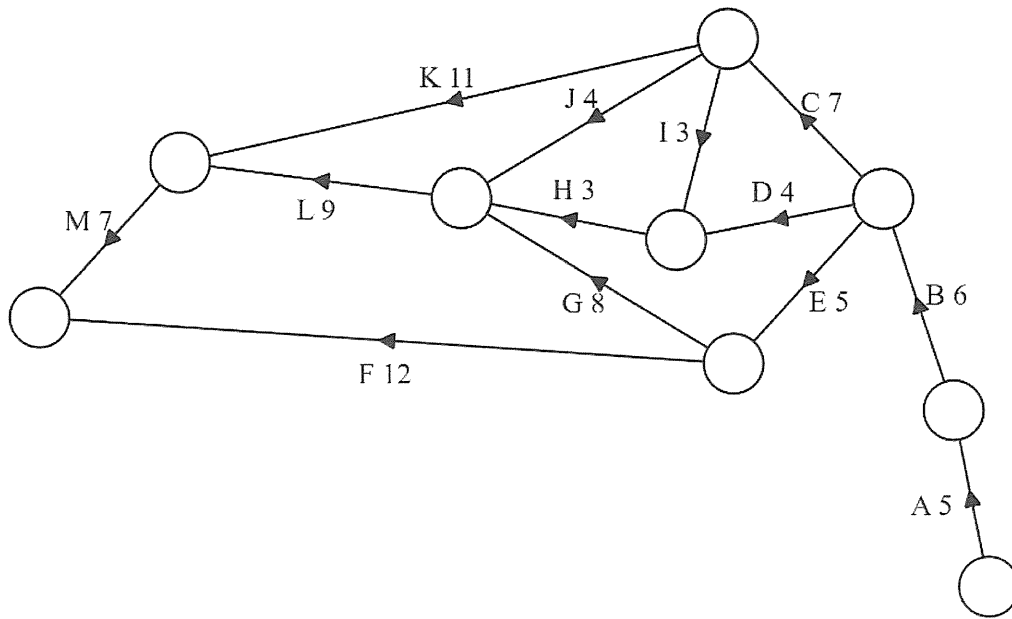
Draw the project network that illustrates the above information.



-1 per error

2. (8 marks)

In the following project network, the times for activities A to M are given in minutes.



(a) State the activities that are on the critical path(s) for this network. [2]

A B C I H L M ✓
A B E G L M ✓

(b) If the project started at 9.00 am, what is the earliest time the project could be completed? [1]

9:40 am

(c) How many minutes can the starting time of Task J be delayed without affecting the time needed to complete the project? Explain. [2]

2mins + Explanation
Eg Float time = 2mins

(d) If the time required to complete Task D becomes 8 minutes longer, how will this affect:

(i) the time needed to complete the project. [1]

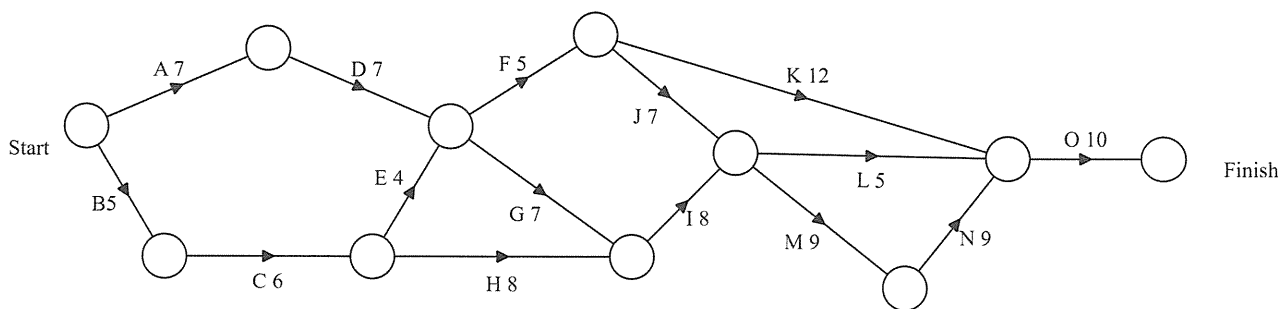
Min completion time is now 42 mins
(2 mins longer)

(ii) the activities along the critical path? [2]

A B D H L M ✓
Now only 1 critical path ✓

3. (8 marks)

In the following project network, the times for activities A – O are given in days.



(a) Complete the table for the project network given above. [2]

Task	Time to complete task	Immediate Predecessors
A	7	-
B	5	-
C	6	B
D	7	A
E	4	C
F	5	D, E
G	7	D, E
H	8	C
I	8	G, H
J	7	F
K	12	F
L	5	J, I
M	9	J, I
N	9	M
O	10	K, L, N

(b) Determine the minimum completion time for the project. [1]

58 days

(c) Identify the critical path(s) [2]

B C E G I M N O

(d) How long can task F be delayed without affecting the completion time? [1]

16 days

(e) Determine the float time for activities D and K. [2]

D = 1 day

K = 16 days

4. (6 marks)

At a school inter-house athletics carnival, a new event is to be introduced. It is the 400 m medley relay. Each house must submit a team of 4 boys, with each boy to participate in one of the 4 legs of the relay. The 4 legs are:

- R: Running
- H: Hopping
- S: Skipping
- B: Running backwards.

One house team consisting of Andrew (A), Ben (B), Corey (C) and Dan (D), have the following personal best times (in seconds) for each leg of the relay.

	R	H	S	B
A	75	95	75	100
B	70	95	80	110
C	85	85	80	95
D	70	80	85	90

(a) Use the Hungarian Algorithm to allow you to decide which is the best way to allocate the boys to the events to make their overall time as short as possible. [5]

Row ↓

$$\begin{bmatrix} 0 & 20 & 0 & 25 \\ 0 & 25 & 10 & 40 \\ 5 & 5 & 0 & 15 \\ 0 & 10 & 15 & 20 \end{bmatrix}$$

Column ↓

$$\begin{bmatrix} 0 & 15 & 0 & 10 \\ 0 & 20 & 10 & 25 \\ 5 & 0 & 0 & 0 \\ 0 & 5 & 15 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 10 & 0 & 5 \\ 0 & 15 & 10 & 20 \\ 10 & 0 & 5 & 0 \\ 0 & 0 & 10 & 0 \end{bmatrix}$$

Allocate :

- Andrew to Skipping
- Ben to Running ~~Bwards~~
- Corey to Hopping
- Dan to Running Bwards

(b) What is that shortest total time? [1]

320 seconds

5. (6 marks)

An orchardist has three types of fruit trees (apples, pears and nectarines) ready for harvesting, and he has 4 fruit pickers, from which he has to choose three. As he is paying the pickers on a dollar per hour basis, he wishes to get as much fruit as possible picked in any hour. The rates (in kg/hour) at which the pickers are able to harvest the fruit are presented in the following table:

	Apples	Pears	Nectarines
Picker 1	10	12	11
Picker 2	12	14	12
Picker 3	9	13	13
Picker 4	8	15	12

Using a method of listing the possible combinations, decide on

(a) State an appropriate method to solve this problem [1]

Hungarian OR Listing all possibilities

(b) Use your method to determine each of the following:

(i) the three pickers who are awarded the work [2]

(ii) the allocation of the pickers to the task and [2]

(iii) the maximum total number of kilograms per hour of fruit that is picked with this combination. [1]

Handwritten matrices showing the solution process:

Initial cost matrix (crossed out):

$$\begin{bmatrix} 10 & 12 & 11 & 0 \\ 12 & 14 & 12 & 0 \\ 9 & 13 & 13 & 0 \\ 8 & 15 & 12 & 0 \end{bmatrix}$$

Reduced cost matrix:

$$\begin{bmatrix} 5 & 3 & 4 & 0 \\ 3 & 1 & 3 & 0 \\ 6 & 2 & 2 & 0 \\ 7 & 0 & 3 & 0 \end{bmatrix}$$

Further reduced cost matrix:

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 2 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 3 & 1 & 0 \end{bmatrix}$$

Final allocation matrix (with boxed zeros):

$$\begin{bmatrix} 2 & 3 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 2 & 0 & 0 \\ 4 & 0 & 1 & 0 \end{bmatrix}$$

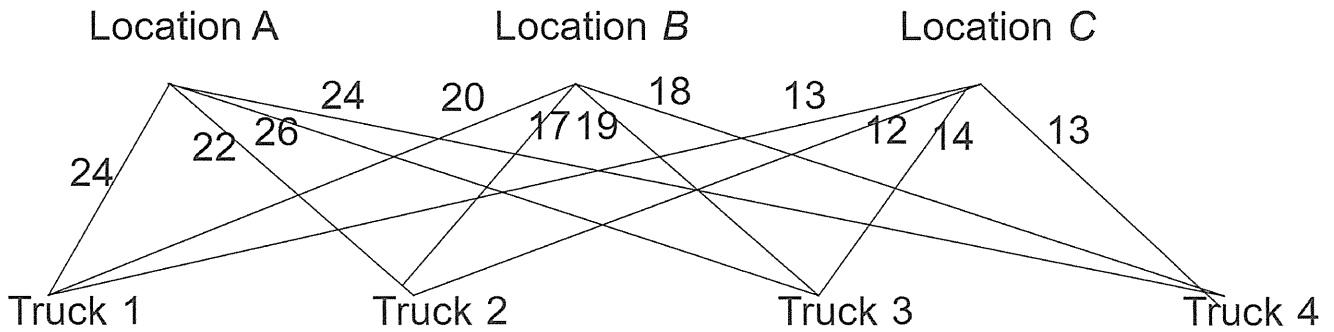
(a) Pickers are : 2, 3 & 4 ✓✓

(b) Allocation : 2 → Apples
 3 → Nectarines ✓✓
 4 → Pears

(c) Max = 40 kg/hr ✓

6. (6 marks)

A delivery company has packages to deliver to three locations, A, B and C, and has four trucks that are available. The graph below shows the distances of each truck from the locations, in kilometers.



(a) Represent the information in the graph as a matrix. [1]

Location

	A	B	C
1	24	20	13
2	22	17	12
3	26	19	14
4	24	18	13

Truck

(b) Show use of the Hungarian algorithm to determine which truck should deliver which package in order that the total distance travelled by the trucks is a minimum, and state what that minimum distance is. *Note: there is additional space on the following page* [4]

$$\begin{bmatrix} 24 & 20 & 13 & 0 \\ 22 & 17 & 12 & 0 \\ 26 & 19 & 14 & 0 \\ 24 & 18 & 13 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & \boxed{0} & 0 \\ \boxed{0} & 0 & 0 & 0 \\ 3 & 1 & 0 & \boxed{0} \\ 1 & \boxed{0} & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 2 & 2 & 0 \\ 2 & 1 & 1 & 0 \end{bmatrix}$$

Truck 1 - C

Truck 2 - A

Truck 4 - B

Total Distance = 53 km

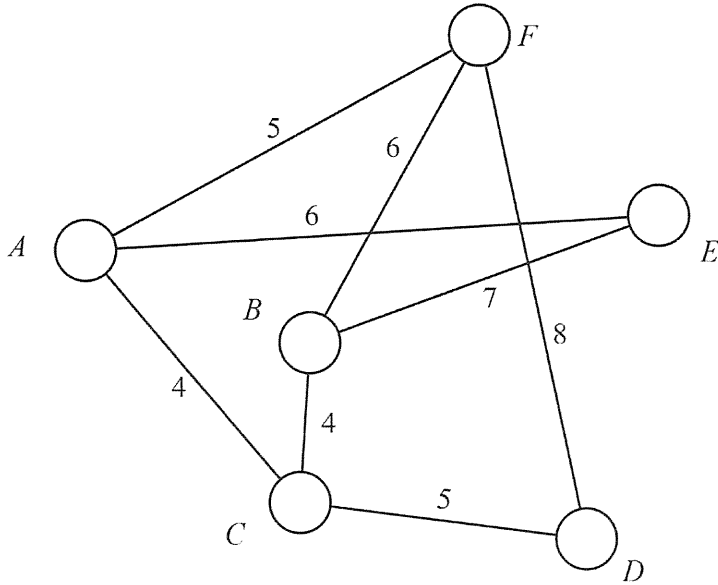
- (c) If the initial distance of truck 3 from location A was reduced by 2 km, explain what effect, if any, this would have on your answer to (b).

[1]

No impact, minimum distance
in that column is still 22.

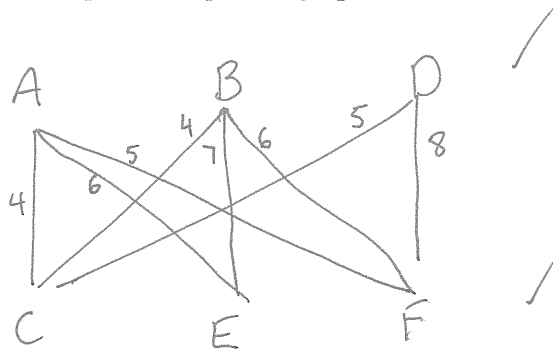
7. (6 marks)

The graph below represents 3 workers and the time it takes them to complete certain tasks.



(a) Re-draw this graph as a bipartite graph

[2]



(b) Determine which person should be allocated to which job in order to complete all jobs in the smallest possible amount of time.

[4]

		C	E	F			
A	[4	6	5]	} Column reduction	
B	[4	7	6]		
D	[5	-	8]		
Row reduction		0	2	1]		
		0	3	2]		
		0	-	3]		

A	to	F	
B	to	E	✓
D	to	C	

[0	1	1]	
[0	-	2]	
[0	0	0]	

Additional Working Space